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**Geometric Series and Exponential Functions Embedded in a Logistics Application**

**Introduction**

This is an application lesson that introduces students to an area of logistics where they determine the operating costs of a transportation contract. The process uses many math skills, especially basic ones; however, it also uses ideas of exponential functions, rational expressions, and geometric series. The lesson is designed to be a final application assignment in the ALGEBRA II unit on sequences and series. Beyond the sequence and series unit, this lesson is designed to give students insight into a possible career path where math is used. Much of the math necessary for this lesson is basic but using the math in the context of a broad problem will be the main challenge. Students will be given an introduction to the field of logistics and some of the possible careers within the field. An expert in the logistics field can be brought in to share their experiences in the logistics field. Then the students will work in partners to complete the application activity.

**Learning Outcomes**

* Students will be able to use exponential functions, geometric sequences, geometric series, and rational expressions within the context of a broader problem.
* Students will come away with an understanding of the logistics field and the job opportunities therein.
* Students will improve their problem solving skills and work to solve math problems in the context of a larger problem.

**Curriculum Alignment**

1. **A-SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
   * In this lesson students will derive the formula of the sum of a finite geometric series and manipulate the formula within the context of the logistics problem to determine the amount of the monthly payments of a loan.
2. **F-BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
   * In this lesson students will create an explicit geometric formula for the equation used to determine the monthly payments of a loan.
3. **A-APR.7** Understand that rational expressions form a system analogous to that of rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
   * In this lesson students will use the rules of the system of rational expressions to verify that two rational equations are identical.

**Classroom Time Required**

The lesson and activity will take three days on the block schedule. You can also build some logistics information into the unit leading up to the lesson.

* Day 1: Introduction to logistics and guest speaker
* Day 2 and 3: Logistics Activity

**Materials Needed**

* A guest speaker: Someone in the logistics field or who teaches logistics at the college level. Using college professors may help to locate a person who does guest lectures at the college level.
* The logistics activity handouts
* An Excel answer template for students to record their work
* Answer keys. If you would like to change any of the numbers in the lesson for any reason, like changes in gas prices, the answer key spreadsheet can be adjusted and the answer key should change automatically.
* Access to computers

**Technology Resources**

Students can use Microsoft Excel and the answer template to input all of their information. This is an optional technology resource. Without it, students will have to do more work by hand. The answer template is set up to complete some of the mathematical calculations. To use the answer template without the mathematical calculations, the formulas can be removed from the answer template.

**Pre-activities**

Information that can be used for the pre-activities can be found in the supplemental information section of the lesson plan.

This lesson is an application activity for the end of the sequence and series unit for ALGEBRA II. The exponents and logarithms unit and rational functions unit should also be completed before this activity is used as skills from those units will be utilized.

The logistics industry and the transportation industry need to be introduced before the lesson. This can be done in many ways. One method would be to have an introduction to logistics day where you introduce the vocabulary used in the activity and begin a discussion about the student’s knowledge of the vocabulary and the logistics/transportation industry (Activities day 1). You can make a handout of the vocabulary words and use the information in the supplemental information section to introduce and talk about the logistics industry. In addition, inviting a guest speaker for ½ - 1 class period is an excellent way to introduce this activity and can be done in correlation with the teacher’s introduction.

Another method would be to integrate the introduction activities throughout the unit so students could have mini 5 – 10 minute lessons on the logistics industry each day. Below is a suggested daily activity for eight day of the unit. Depending on the length of the unit you could expand on these activities or remove/change some of the activities. The activities below are not step by step planned lesson, rather they are suggestions for short, 5 – 10 minute activities. Many of these activities could be short readings followed by a group discussion or asking probing questions to assess students prior knowledge.

1. Have students answer the question “What is logistics” either in a large group or in smaller groups. See what the student come up with for answers.
2. Introduce the field of logistics and transportation using the information in the supplemental information section of the lesson plan. Compare this information to what the students came up with the day before. Talk about the difference between the logistics and transportation industry and the way logistics is used in every industry.
3. Ask students what types of jobs they think are necessary in the field of logistics. See if they can come up with things like analysts, marketing, lawyers, computer programmers, truckers, secretaries, and CEOs. Ask leading questions to elicit responses such as “How will a transportation company find business?” or “How are companies held to the contracts they sign?”
4. Ask students what they think an analyst does. Engage in a conversation. Focus on the word analyze if necessary and focus on why transportation companies would need analysts to determine operating costs.
5. If transportation companies need analysts, why would other companies need logistics analysts? If you own a business that makes some product that sells in thousands of stores across the country, why would you need a transportation analyst?
6. Show the students the scenario of a company without an analyst provided in the supplemental materials section. Discuss why it is important for businesses to know their profitability.
7. Discuss why knowing profitability is important in other areas, not just logistics. For example, what if you wanted to open a restaurant, what would you need to know to be profitable? Another example could be that you invent a product and want to manufacture and market your product, what would you need to know to be profitable? Answers in this scenario should focus on overhead costs like rent, cost of manufacturing, cost of shipping, payroll; and the price point for what you are selling. If you open a burger joint and you make $0.25 per burger can you sell enough burgers to cover all of your costs?
8. Vocabulary introduction – discuss the meaning of each of the logistics vocabulary words. You could give the list of words to your students or ask them to think about what each definition is. Some of the definitions like profitability should be known to students already so they could come up with some good definitions. Others like PUD would need to be explained fully. Attached is a vocabulary activity sheet that has the vocabulary words with the definitions, a vocabulary matching activity, and a vocabulary definition writing activity.

**Activities**

**Day 1 – Logistics Introduction (pre-activity)**

* Introduce Logistics using the supplemental information and using the activities listed above in the pre-activities section.
* Introduce Logistics vocabulary: Distribution Center, Dispatch Center, Hub, Pallet, Operating Cost, Bid, Contract, Profitability, Depreciation, PUD, Line-haul. Use one of the attached vocabulary activities.
* Invite a guest speaker from the logistics field, preferably a logistics professor or college guest speaker who knows the field and has practice relaying the information to students.

**Day 2 – Logistics Activity**

* Put students in groups of 2.
* Hand out the logistics activity
* Read the scenario together with your students (5 minutes)
* Discuss the “Operating Cost Data”, “Truck Route Information”, and “Determining Operating Costs” sections of the activity with your students to prepare them to get started. Discuss what each section is but not the specifics within each section. The operating cost data is where you find general values to be used in your calculations. The truck route information is where you find the specific information pertaining to this job. The determining operating costs section is where you bring the first two sections together to find the operating costs for this job. I have found that when I discuss specifics too much my students become dependent upon my help and have trouble finding the information independently. (5-10 minutes)
* Ask students to find the key vocabulary from Day 1 and identify other words that they do not know. (5-10 minutes)
* Have students begin the logistics activity. (50-60 minutes)
* Circulate around the room and monitor students. Make sure their mathematical calculations are correct. If a group is struggling with the mathematical calculations, ask them leading questions to get them back on track. If students are getting off task use proximity management. Move closer to the group, and if necessary, engage in discussion questions about where they currently are in the activity to draw their attention back to their task. Monitor groups for areas where many groups are struggling with the same math problem or the same section of instructions. If many groups are struggling refocus whole class attention and address the specific area of concern. Try to ask leading questions. For example: “In the PUD section, what are you supposed to calculate? Do you have the information for hours? Where can you find that information? How about the information for pay per hours? What do you do with this information?” Try not to give answer, rather ask directed questions and have students answer these questions. If it happens for more than one section try to get the students or a student to ask the directed questions to help them realize that if they ask themselves the questions they can figure out how to do the problem on their own.
* Have students complete the end of class questions located at the end of the activity. These questions can serve as an informal assessment for the first day of the activity. (10-15 minutes)
* Collect the activities and end of class questions from your students to look over them and identify points of discussion for day 2. Look to make sure students could do both the mileage and line haul sections and if any students made it to the equipment costs section, make sure they are on the right track. Prepare points of discussion based on any of the activities that students struggled with mathematically. Identify groups that are behind or are getting incorrect values when they are doing their mathematical calculations. When you circulate during day 3, make sure you focus on these groups and help them maintain a good pace and identify mistakes they made from the first day.

**Day 3 – Logistics Activity**

* Pass out the logistics activity
* After looking over the work from day 2, plan for a brief introduction for day 3 that addresses any concerns you may have about the work that was done during the first day. You may need to go over some of the math involved that students had trouble with during day 1 or review geometric sequence and series to prepare students for that section of the activity. (10-20 minutes)
* Address student questions and concerns. (5-10 minutes)
* Have students continue working on the activity (60-70 minutes)
* Continue to monitor students. In the beginning focus on groups who had some mistakes from what they did on day 2, and groups that are behind the pace of their classmates.
* For groups that finish early, see the modification section. You could also pair these students up with groups that are behind for peer tutoring and assistance.
* At the end of the class collect and grade the activity. If certain students need more time and assistance, make arrangements for groups to come before or after school to finish. Call parents to help remind the students when to show up.

**Assessment**

The application can be graded based on the project rubric. Each section will be graded based on the correct mathematical answer. Partial credit can be given for students who were on the right track but made a mathematical error. Each part of the problem will be graded for correctness. Errors will not be compounded so a mistake on any of the first 5 sections will not be cause for students losing points in the later sections. The end of class activity from day 2 is not a part of this rubric and can be graded as a separate assignment.

**Modifications**

I have divided up all of the activities within the application into their basic components. For students who need the maximum amount of direction a teacher could use all of these individual components. Based on your student’s ability levels and proficiency with solving problems using ingenuity you can remove some of those guided components to allow students to work through the entire, or at least large parts of the, problem on their own. I plan on allowing my students to work independently on all of the activities except for the one dealing with geometric sequence and series. Geometric sequence and series is the curriculum alignment so that part will be more guided. In addition you can make the problem more complicated by introducing the idea of a dead head run and having students create a cost analysis for the scenario if deadheading in one direction is necessary. A dead head run is a run in which there is no cargo on the return trip. In the trucking industry companies try to avoid dead head runs because they greatly increase the cost of the run which increases the amount of their bid. Dead head runs also limit a company’s profits because they do not have a second job that they can bring in more money. Even though these types of runs are avoided, many dead head runs do happen because of unavoidable circumstances.

**Alternative Assessments**

There is an informal assessment built in at the end of day 2 as an end of class activity.

**Supplemental Information**

Logistics is the science of moving resources. This could be internal movement, moving supplies from one location to another within a building or from one location to another location. This could also be an external movement, shipping product from the manufacturer to the store or being a company that does the transportation for another company. The main ways that products are moved externally are by truck, train, boat, or plane. We have all seen examples of these different types of transportation. We see trucks every day on the roads. Most trains that we see are not passenger trains but are instead cargo trains with massive containers that store products of all different types for shipment. Boats are used to transport across seas. Planes are used to ship product with great speed. Transportation companies such as Fed-Ex and UPS have their own hubs at airports across the world. The interconnectedness of all of these transportation industries is very details. Each transportation industry has its own rules, guidelines, and challenges. There are pros and cons to using each type of transportation to move product and often more than one type of transportation is used to get a product from point a to point b.

There are several different fields of logistics from procurement logistics, to distribution logistics, to global or domestic logistics. The logistics industry is a massive industry that has jobs in every field. Basically, whatever your skill set, if you want to pursue logistics as a career path there is a place for you.

For the purposes of this unit the focus is on the math of a logistical analyst. An analyst is a person who analyzes the cost of transporting goods. Analysts can be employed at all levels within the logistics field. Transportation companies need analysts to determine how much money each job will cost so that bids can be created. Companies that retain the services of transportation companies need analysts to determine how much a job will cost so they can analyze bids that are coming in for certain jobs. There are even logistical analysis companies that work as middle men and do the analysis for both transportation companies and the companies that want to hire transportation companies.

Talking to people within the industry, there are several companies who try to work without analysts and they don’t survive. A common scenario is one where a company wants to move product and asks for bids from several transportation companies. They take the bid from the lowest bidder. The problem is that that lowest bidder did not know what the operating cost of the transportation run would be so they bid at a price that not only didn’t make a profit but actually lost money. The transportation company tries to fulfill their contract for as long as they can, while losing money the entire time, until they go bankrupt. This in not just a problem for the transportation company but it is also a problem for the hiring company. When the transportation company fails to fulfill its contract the hiring company has no way to ship its merchandise which causes a problem with sales, and they have to go through the whole process of getting bids again for the same job. Analysts are needed at the transportation company so they don’t underbid and lose profitability. Analysts are also needed at the hiring company to determine which bids are reasonable. Low bids sound great but will often go unfulfilled costing the company money and high bids hurt the profit margin of the company. These same logistical analyst skills are utilized or underutilized in many other businesses. Many businesses open for a short time and then close again because their profitability was never analyzed. Whatever they are attempting to sell has a price point that is too low and even when some of these companies appear to be doing well because they are busy all of the time, they still do not have enough customers to stay in business because they are undervaluing their product. Many of us can also think of examples of jobs, especially in construction, that were started and then ran into trouble because the lowest bidder for the job could not follow through on what was promised or because the lowest bidder was that low because the workers it employed were not qualified to do the work. Work on these jobs may stop completely because the lowest bidder abandoned the job, the work might slow to a crawl as the lowest bidder attempts to renegotiate, or the work may need to be redone because the craftsmanship was of such low quality.

Being an analyst is a math intensive field but all of the math that is used for analysis can be done around the Algebra II level. Some of the harder math that will be done in this lesson is beyond the scope of most analysts and can be done with analyst software. What is more important is learning how the transportation industry works and where all of the costs occur so operating costs can be determined.

Students who are interested in pursuing a career in logistics have education options for two year associates degrees all the way up through master’s degrees in certain logistical disciplines. Students can earn degrees in fields like “Supply Chain, Logistics, and Transportation Management” and “Global Logistics Technology”. Many people who go into the logistics field do so as a second career because they didn’t discover this career pathway until they were older and saw the logistics industry from whatever job they had originally. Many younger people think of logistics as simply the trucking industry and they think that the only job in the industry is that of a trucker. Being a trucker is one of the many important jobs within the logistics industry but there is so much more depth to the industry than most people know. Transportation companies employ people from almost every field including fields like, business managers, lawyers, analysts, truck drivers, dock workers, and the list goes on and on. The transportation industry is also a growing industry which offers people a stable work environment.

**Critical Vocabulary**

* Distribution Center – A warehouse which houses products to be distributed to retailers.
* Dispatch Center – The location the trucks are housed at and start their runs from. A dispatch center often has its own warehouse and office area and is often the central office of a transportation company.
* Hub – Central location for a company. Nomadic transportation has a hub in Greensboro and one in Minneapolis which means that they have a business center in each location
* Pallet – A flat, typically wooden transportation structure for transporting goods which is designed to be moved by a fork lift or a pallet jack. Pallets are typically around 4’ x 3.5’ and can be stacked several feet high with product which is typically shrink wrapped to hold the product in place.
* Operating Cost – The amount of money it costs to complete a job. The operating cost includes all of the expenses necessary to complete the job. The profit made from a job must be added to the operating cost of the job when a bid is made.
* Bid – When a company is looking to transport material they will often contact several transportation companies to get bids. A bid is the amount of money that a transportation company will do a job for. A bid can be static, meaning that there is one price for the job, or it can be variable, meaning that the price could change due to certain factors like gas prices. For a long term contract, such as this one there are often variable costs included in the bid. Once a company has received all of the bids it must choose which transportation company to work with.
* Contract – A written document between two companies stating the terms of the working relationship. The contract will have all of the information from the winning bid written into it.
* Profitability – A measure of operating cost and profit margin. The profitability of a contract determines how much money can be made from a contract. Not every profitable venture has good profitability. If a contract has minimal profit but uses many resources that could be utilized in other, more profitable ways then its profitability is low. This contract with utilize major resources so it is necessary to have higher profitability to take on the job.
* Depreciation – The decrease in value of assets. The truck and trailer that would be purchased for this run are considered company assets. As these assets age, their value decreases. In a logistics model, the depreciation time is the estimated life of the vehicle. A tractor has a 5 year depreciation meaning that it is expected to last for 5 years. The cost of the tractor is then split among those 5 years so the cost of depreciation for each run can be figured into the bid.
* PUD – Stands for pick-up and delivery. This is the time between when the truck leaves to pick up a product and when the truck starts its line haul. This is also the time after the truck gets to its destination until the truck is back at the dispatch center.
* Line Haul – The line haul is the main run of the truck. The line haul starts after everything is loading onto the truck for transport and ends when the truck reaches the delivery site.

**Websites**

* <http://www.nccgl.com/>
  + The North Carolina Center for Global Logistics
  + “The North Carolina Center for Global Logistics is a resource provider of logistics education and training furnished by a collaborative arrangement with community colleges, four-year colleges and universities”. This quote was taken from the NCCGL website. The NCCGL is a local organization that focuses on logistics education and logistics workforce development.
* <http://en.wikipedia.org/wiki/Logistics>
  + Wikipedia
  + A general overview of logistics that can be used to help prepare for the logistics lesson. Yes it is Wikipedia but the information is good and easy to understand.
* <http://www.sole.org/>
  + The International Society of Logistics
  + A logistics society that can be contacted and used as a resource for the lesson.
* <http://www.astl.org/i4a/pages/index.cfm?pageid=1>
  + American Society of Transportation and Logistics
  + A logistics society that can be contacted and used as a resource for the lesson.
* <http://www.logisticsmgmt.com/>
  + Logistics Management Blog
  + A logistics blog that has many interesting articles that can be used as supplemental material. One of the blog posts could be used as supplemental material reading for a pre-activity.
* <http://www.alanaid.org/>
  + American Logistics Aid Network
  + A nonprofit organization that works with the logistics of aid efforts. As the website says “Engaging the supply chain community to support humanitarian relief efforts”. This website could be used as a resource to expand this lesson into a possible service learning opportunity.
* <http://www.uncg.edu/bae/online/Post_Bac_Cert_Supply_Chain.htm>
  + University of North Carolina - Greensboro
  + An example of a logistics program at UNCG that could be showcased to students as an example of a possible logistics program. This is a graduate level program. Similar programs can be found at most schools in North Carolina.

**Comments**

This lesson was developed as a product of my work as a Kenan Fellow working with the NC Center for Global Logistics with many thanks to my mentor Karl Robinson of R & R Transportation and the NC Center for Global Logistics and to Nemiah Bryant, a college professor at UNCG and GTCC and a Risk Assessment Officer at BB&T Bank, both of whom were instrumental in the creation of this lesson.

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